References

[1]Ousley, G., "Execution Phase Project Plan for Active Magnetospheric Particle Tracer Explorer (AMPTE)," NASA, Aug. 1980.

[2]Thompson, R., and Neal, G., "Active Magnetospheric Particle Tracer Explorer (AMPTE) Engineering Data Simulator Description and User's Guide," Computer Sciences Corporation, CSC/TM-81/6069, April 1981.

[3]Neal, G., and Thompson, R., "Inflight Estimation of Magnetometer Biases for the AMPTE Mission," Computer Sciences Corporation, CS/TM-81/6070, May 1981.

[4]Wertz, J.R. (ed.), *Spacecraft Attitude Determination and Control,* D. Reidel Publishing Company, Dordrecht, The Netherlands, 1978, pp. 329-330.

[5]Werking, R.D., "A Generalized Technique for Using Cones and Dthedral Angles in Attitude Determination, NASA X-581-73-292, Sept. 1973.

[6]Shuster, M.D., "Efficient Algorithms for Spin-Axis Attitude Estimation," Flight Mechanics/Estimation Theory Symposium, NASA Goddard Space Flight Center, Greenbelt, Md., Oct. 1981.

Modification of Pitching Stability by an Atmospheric Wave

George Chimonas*
Georgia Institute of Technology, Atlanta, Georgia

Introduction

ATMOSPHERIC disturbances can modify the flight characteristics of aircraft. Extreme examples of this are the destructive consequences of downbursts[1] and the disruptive effects of CAT (clear air turbulence). Wind shear and turbulence are particularly unwelcome in the phases of take-off and landing, where they may critically interfere with aircraft control.[2,3,4,5] In this Note a related effect is demonstrated—the modification of the high-frequency oscillation when an aircraft encounters a coherent wave disturbance. The effect is most noticable when the wave frequency, seen from the aircraft, is twice the frequency of the free pitching mode. The interaction modifies the damping rate of the motion and leads to excitation of all harmonics of the pitching frequency. The effect is something of a curiosity, in that while it may enhance aircraft vibration a little, it should not lead to instability unless the aircraft is already dangerously underdamped.

The atmospheric waves are internal gravity waves of the gravity shear or Kelvin-Helmholtz variety.[6] It is relatively common for aircraft to encounter such waves, since flight paths often coincide with the jet streams, favored locations for shear waves. However, the more extreme instances of wave activity are likely to be avoided, since they coincide with servere CAT.[7]

Model Calculation

The calculation is an elementary study of the longitudinal pitching mode of an aircraft flying with fixed controls. Severe approximations to the dynamics and total absence of any control corrections make this calculation "preliminary" at best. Its advantage is simplicity, in both presentation and interpretation.

Submitted Feb. 1, 1983; revision received June 7, 1983.

*Professor, Department of Geophysical Sciences.

In its simplest form, the "high frequency" mode is defined by the aircraft behavior in a uniform atmosphere (e.g. Seckel[8]). It has also been studied in the presence of a horizontally homogeneous vertical shear.[5] The present study allows periodic variations in the wind field. Aircraft response has been obtained in several models with various degrees of complexity. The most complex has been a two-air-foil rigid craft in free longitudinal motion with coupled plunging and pitching. But the essential result is easily illustrated by isolating the perturbation moment generated by the lift of the tail air foil. This most simple model produces essentially the same pitching behavior as the most complicated formulation. Perhaps this is not surprising in that it reflects the dominant role of lift at the tail air foil in all matters of pitching stability. The simplest formulation is presented in this Note.

Formulation

Figure 1 defines the geometry of the system. The aircraft is trimmed for level flight at constant air speed S when it encounters a periodic wind perturbation. Changes in the magnitude and direction of the tail lift, L, induce pitching through angle θ about the center of gravity of the aircraft:

$$I\ddot{\theta} = \mathrm{d}L\cos(\phi-\theta) - M_0 \tag{1}$$

I is the moment of inertia of the craft and d the effective moment arm of the tail, about the center of gravity. The moment of the tail lift in trimmed level flight, M_0, is balanced by other torques whose perturbations are normally small compared with the right hand side of Eq. (1).

In this model, the center of gravity remains in uniform horizontal motion with constant ground speed. Air speed relative to the tail, V, changes because of perturbations (u,w) in the wind, and the tail rotation velocity ($\mathrm{d}\dot{\theta}\sin\theta$, $\mathrm{d}\dot{\theta}\cos\theta$). An analogous separation in the dynamics was used by Biggers[9] in his study of the flapping stability of helicopter rotors. After assuming a constant forwards speed for the axis of the rotor system, Biggers obtained an equation with periodic coefficients for the motion of the rotor blades. In this sense also his study resembles the calculation presented below.

Equation (1) is simplified by assuming that the lifting coefficient, $c(\alpha)$, varies linearly with changes in angle of attack α about a mean setting α_0. The dynamics can then be set up with the relations

$$L = \tfrac{1}{2}\rho A\{c_0 + c_l(\alpha-\alpha_0)\}V^2 \tag{2}$$

$$\alpha - \alpha_0 = \phi - \theta \tag{3}$$

$$M_0 = \tfrac{1}{2}\mathrm{d}\rho A c_0 S^2 \tag{4}$$

$$\phi = \tan^{-1}\{(w - \mathrm{d}\dot{\theta}\cos\theta)/(S + \mathrm{d}\dot{\theta}\sin\theta - u)\} \tag{5}$$

$$V^2 = (S + \mathrm{d}\dot{\theta}\sin\theta - u)^2 + (\mathrm{d}\dot{\theta}\cos\theta - w)^2 \tag{6}$$

Combining Eqs. (1)-(4)

$$I\ddot{\theta} = \tfrac{1}{2}\mathrm{d}A\{c_0(V^2\cos(\phi-\theta) - S^2) + c_l(\phi-\theta)V^2\} \tag{7}$$

Now two, distinct, small amplitude approximations are applied. First, all equations are linearized in θ, the disturbance to the pitch. This is the standard approximation of "infinitessimal" stability theory. Second, the wave amplitude only appears as corrections of order u/S and w/S in expression that are of order 1. Since these wave terms are relatively small, all equations are further linearized in these amplitudes. It must be noted that in this procedure response terms of order $\theta\ (u/S)$ are retained while formally larger forcing terms of order $(u/S)\cdot(u/S)$ are discarded. This approach cannot be rigorously justified. But to the extent that stability can be determined from the behavior of the homogeneous sytem alone, as is attempted below, these forcing terms need not be considered anyway. It would, however, be very interesting to see if a fully nonlinear calculation retained the features deduced from the present linearized approach.

These linearizations reduce Eqs. (5)-(7) to

$$\ddot{\theta}+2\zeta\omega_0(1-u/S-5c_0w/2c_1S)\dot{\theta} + \omega_0^2(1-2u/S-c_0w/c_1S)\theta=\omega_0^2(w/S-2c_0u/c_1S) \quad (8)$$

where ω_0 and ζ are found to be

$$2\zeta\omega_0=M_0\mathrm{d}c_1/ISc_0 \quad (9)$$

$$\omega_0^2=M_0c_1/Ic_0 \quad (10)$$

The notation has been chosen to coincide with the properties of the homogeneous wind problem. When the wave components u and w are set equal to zero in Eq. (8), the system reduces to the damped harmonic oscillatory equation that is the most elementary description of the "high frequency" mode. The natural frequency ω_0 and the damping ratio ζ of the mode can be assigned values that are observed in actual flight configurations.

Aircraft Behavior

The solution of Eq. (8) can be partitioned into a response forced by the terms on the right hand side, and the general solution of the homogeneous equation. The first part is certainly interesting in its own right, but this study is concerned with the second part, θ_H:

$$\ddot{\theta}_H+2\zeta\omega_0(1-u/S-5c_0w/2c_1S)\dot{\theta}_H + \omega_0^2(1-2u/S-c_0w/c_1S)\theta_H=0 \quad (11)$$

Assume a sinusoidal form for the wave disturbances

$$u/S=A\cos(\omega t) \quad (12)$$

$$c_0w/c_1S=B\cos(\omega t+\phi_1) \quad (13)$$

In Eqs. (12) and (13), ω is the frequency of the wave as seen from the aircraft. Now transform to canonical form

$$\theta_H=y\exp-\zeta\omega_0\{t+A\sin(\omega t)/\omega+5B\sin(\omega t+\phi_1)/2\omega\} \quad (14)$$

$$\ddot{y}+\{\omega_0^2(1-\zeta^2)-E\cos(\omega t+\phi_2)+\epsilon^2\}y=0 \quad (15)$$

where

$$E\cos(\omega t+\phi_2)=2\omega_0^2(1-\zeta^2)A\cos(\omega t) + \omega_0^2(1-5\zeta^2)B\cos(\omega t+\phi_1) + \zeta\omega\omega_0\{A\sin(\omega t)+5B\sin(\omega t+\phi_1)/2\} \quad (16)$$

$$\epsilon^2=\zeta^2\omega_0^2\{A\cos(\omega t)+5B\cos(\omega t+\phi_1)/2\}^2 \quad (17)$$

Now ϵ^2 is second order in the wave amptitudes. Terms of this order were discarded in the linearization procedure, and to maintain consistency the term $\epsilon^2 y$ should be dropped from Eq. (15) giving

$$\ddot{y}+\{\omega_0^2(1-\zeta^2)-E\cos(\omega t+\phi_2^2)\}y=0 \quad (18)$$

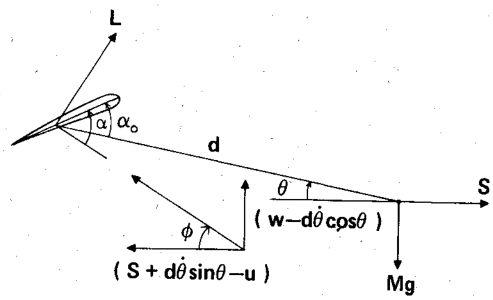

Fig. 1 Flight geometry showing the undisturbed velocity S, the wave field (u,w) and the induced pitching θ.

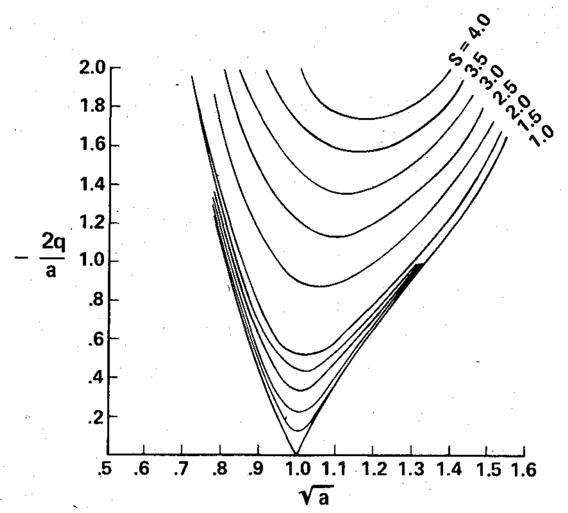

Fig. 2 Contours of the growth rate μ in the second unstable region of (q,a) space. The solid lines are contours of $S=\exp(\pi\mu)$. From Blanch.[10]

This is Mathieu's equation.[10] To put it into standard notation, define

$$2z=\omega t+\phi_2 \quad (19)$$

$$a=4\omega_0^2(1-\zeta^2)/\omega^2 \quad (20)$$

$$q=2E/\omega^2 \quad (21)$$

This leads to the standard equation, and its series solutions

$$\mathrm{d}^2y/\mathrm{d}z^2+(a-2q\cos 2z)y=0 \quad (22)$$

$$y_\pm=A_\pm\exp\{\pm\mu z\}\sum_{k=-\infty}^{\infty}C_{2k}\exp\{\pm i2kz\} \quad (23)$$

The parameter μ may be either real or imaginary, depending on the values (a,q). The high frequency stability is modified by real μ. In the original notation:

$$\theta_H=\exp\{-\zeta\omega_0t+A\sin(\omega t)/\omega+5B\sin(\omega t+\phi_1)/2\omega\} \times\left[A_+\exp\{\mu\omega t/2\}\sum_{k=-\infty}^{\infty}C_{2k}\exp\{ik\omega t\} + A_-\exp\{-\mu\omega t/2\sum_{k=-\infty}^{\infty}C_{2k}\exp\{-k\omega t\}\right] \quad (24)$$

Even though the wave is purely sinusoidal, the craft motion has a very complex spectral form. It is no longer reasonable to define a damping ratio, but θ_H does have obvious exponential decay terms. It can be factored into two parts, one with an overall decay $\exp\{-(\zeta\omega_0+\mu\omega/2)t\}$, the other with $\exp\{-(\zeta\omega_0-\mu\omega/2)t\}$.

This modification of the damping will only be of consequence when $\mu\omega/2$ is at least comparable with $\zeta\omega_0$. Usually this will not be the case. It appears that aircraft with reasonably damped high frequency modes will remain stable for realistic amplitudes of the wave. However, ζ may be very small in special circumstances for modern super-performance aircraft flying at high altitudes. For then $\zeta \to 0$. It should also be noted that damping may be reduced by effects that can occur simultaneously with the wave disturbance studied here. Thus Curtiss,[11] in another study involving aircraft performance governed by equations with time dependent coefficients, has shown that the damping ζ is reduced during deceleration of a craft. Accordingly, very light damping, while unusual, may be encountered in practice.

Pursuing this limit, it is found that the greatest destabilization occurs in the so-called "second unstable region" of the (a,q) space. This is a region of real μ localized about values of a^2 near unity in the notation of Eq. (22) (i.e., a wave frequency ω that is about twice the oscillation frequency ω_0 of the mode). The region is shown in Fig. 2. Only rather small vaues of $2q/a$ are expected, so a good approximation to E is obtained with

$$E \simeq \{4\omega_0^4(1-\zeta^2)^2+\zeta^2\omega^2\omega_0^2\}^{1/2}u/S \simeq 2\omega_0^2 u/S \tag{25}$$

$$|2q/a| \simeq |2u/S| \ll 1 \tag{26}$$

Air speed S will be close to the speed of sound, and the modal period will not exceed a few seconds. Correspondingly, $|2u/S|$ probably cannot exceed 0.1. Examination of Fig. 2 shows that μ will not be much bigger than 0.03. Examination of Eq. (26), for $\omega=2\omega_0$, shows that the craft motion will only be unstable if $\zeta<\mu$, i.e., if $\zeta<0.03$, which is an abnormally small damping ratio.

In conclusion, this mechanism, which is a parametric destabilization of the higher frequency longitudinal mode, is theroretically possible but unlikely in practice. It could of course be induced by loading the craft to have an abnormally long period of pitching but this is hardly a standard procedure.

References

[1] Fujita, T.T., and Caracena, F., "An Analysis of Three Weather-Related Aircraft Accidents," *Bulletin of the American Meteorological Society*, Vol. 58, Nov. 1977, pp. 1164-1181.

[2] Frost, W., and Crosby, B., "Investigations of Simulated Aircraft Flight Through Thunderstorm Outflows," NASA 3052, Sept. 1978.

[3] McCarthy, J., Blick, E.F., and Bensch, R.R., "A Spectral Analysis of Thunderstorm Turbulence and Jet Transport Landing Performance," *Conference on Atmospheric Environment of Aerospace Systems and Applied Meteorology*, American Meteorological Society, Nov. 1978, pp. 1-62.

[4] McCarthy, J., Blick, E.F., and Bensch, R.R., "Jet Transport Performance in Thunderstorm and Wind Shear Conditions," NASA 3207, Dec. 1979.

[5] Sherman, W.L., "Theoretical Study of the Effect of Wind Velocity Gradients on Longitudinal Stability and Control in Climbing and Level Flight," NASA Techncial paper 1332, 1978.

[6] Lalas, D.P., and Einaudi, F., "On the Characteristics of Gravity Waves Generated by Atmospheric Shear Layers," *Journal of the Atmospehric Sciences*, Vol. 33, July 1976, pp. 1248-1259.

[7] Metcalf, J.I., and Atlas, D., "Microscale Ordered Motions and Atmospheric Structure Associated with Thin Echo Layers in Stably Straitified Zones," *Boundary Layer Meteorology*, Vol. 4, Jan. 1973, pp. 7-35.

[8] Sekel, E., "Characteristic Longitudinal Transients—Restricted Degrees of Freedom," *Stability and Control of Airplanes and Helicopters*, Second Printing, Academic Press, New York, 1968, pp. 178-183.

[9] Biggers, J.C., "Some Approximations ot the Flapping Stability of Helicopter Rotors," *AHS/NASA-Ames Specialists' Meeting on Rotorcraft Dynamics*, Moffet Field, Calif., Feb. 1974.

[10] Blanch, G., "Mathieu Functions," *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Ninth Printing, Dover Publications, New York, 1970, pp. 722-743.

[11] Curtiss, H.C., "An Analytical Study of the Dynamics of Aircraft in Unsteady Flight," USAAVLABS Technical Rept. 65-48, 1965.

Stochastic Simulation Using Covariance Techniques: Modular Program Package for Nonlinear Missile Guidance

R. Froriep* and D. Joost†

DFVLR Institute for Flight System Dynamics

Wessling, Federal Republic of Germany

Introduction

SIMULATIONS are often necessary to assess system performance. In this way, the influence of effects that were not taken into account during system design can be examined. Two of the more important of these effects are nonlinearities and stochastic disturbances. For judging system performance generally, mean values and covariances are the most important. Traditionally, nonlinear systems are simulated by applying *Monte Carlo techniques*. Let the nonlinear system be described by

$$\dot{x}(t)=f[x(t),t]+B(t)w(t) \tag{1}$$

driven by a random process $w(t)$ and with random initial conditions. Using shaping filters, we can assume that $w(t)$ is white noise. To obtain statistically meaningful results, a great number of sample responses are generally needed, leading to a high computational burden. The results are, e.g., mean and standard deviation of system states for specific instants of time.

Covariance techniques are methods for directly calculating analytical approximations for the means and the covariances of system states as functions of time. Instead of calculating many sample responses, Eq. (1) is linearized around its mean value and the resulting (nonlinear) system of equations for the means and the covariances is solved numerically only once. The computing time is thus reduced. For the linearization of Eq. (1), two methods can be applied: Taylor series and statistical linearization. In contrast to the CADET-covariance technique,[3] where the whole of Eq. (1) is statistically linearized, we apply a combination of both methods, which is mathematically easier to use and more efficient to compute. Moreover, this combination allows easy design of modular computer programs structured according to physical subsystems.

Covariance Technique for Stochastic Simulation

We linearize the nonlinear stochastic differential equation (1) with the Taylor series expansion around its actual mean value. But we exclude from the Taylor linearization those points in the mathematical model [Eq. (1)] for which the

Received June 7, 1983.

*Diplomate in Engineering.

†Diplomate in Mathematics.